MG26018 Simulation Modeling and Analysis

Sino-US Global Logistics Institute
Shanghai Jiao Tong University

Fall 2019

Assignment 3

Due Date: November 28, 6:00 PM

Instruction

(a) You can answer in English or Chinese or both.

(b) Show enough intermediate steps.

(c) Write the answers independently.

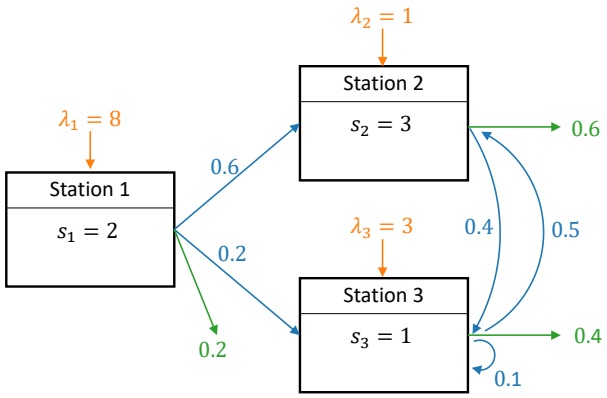
(d) **Send the electronic version of your answers, together with the final Arena `doe` file, to shenhaihui@sjtu.edu.cn before deadline; NO late submission will be accepted.**

Consider the queueing network in the following figure. For $i = 1, 2, 3$, station i has external arrival following Poisson process with rate λ_i and s_i servers, whose values are as shown. A server in station 1 has service time following exponential distribution with rate 6. A server in station 2 has service time following Erlang distribution $\text{Erl}(k, \lambda)$ whose density is

$$f(x) = \frac{\lambda^k x^{k-1} e^{-\lambda x}}{(k-1)!}, \ x \geq 0,$$

and $k = 2, \lambda = 8$. A server in station 3 has service time following Weibull distribution $\text{Weib}(\alpha, \beta)$ whose density is

$$f(x) = \frac{\alpha}{\beta^\alpha} x^{\alpha-1} e^{-(x/\beta)^\alpha}, \ x \geq 0,$$



and $\alpha = 1/2, \beta = 1/24$. The unit of all the above time is **minute**. Each station has a single queue and unlimited waiting space. The routing probability to other stations and to departure are as shown.

Suppose this system opens at 9 AM (in an empty state) and closes at 5 PM. We want to conduct a simulation in Arena to estimate the expected average waiting line length in each station, and to evaluate some potential improvement. For simplicity, we just let our simulation stop exactly at 5 PM and ignore the remaining customers.

Caution: Pay attention to the different distribution parameter definition in Arena! (Perhaps other softwares also have such issue.) See more details in the supplementary reading: Elements of Probability. Surprisingly, this book chapter has more typos than I expected. I have just further fixed some serious typos. Please ***check the latest version*** *on the course website. Moreover, I also have uploaded another piece of material: Arena's Probability Distributions, which should be a more reliable reference for clarifying the distribution parameters in Arena.*

Question 1 (5 points)

Which type does this desired simulation belong to? Terminating or nonterminating? Why?

Question 2 ($5 + 20 + 10 = 35$ points)

Suppose the model has been built in Arena. In order to do the verification, i.e., to make sure that there is no bug, at least for the logical part, we want to compare the simulation results with some analytical results. For such purpose, we *temporally* and *artificially* assume that the service time in stations 2 and 3 also follows exponential distribution, which makes the problem a Jackson queueing network. More specifically, let service time in station 2 follow exponential distribution with rate 4, and service time in station 3 follow exponential distribution with rate 12.

(1) In order to do comparison with the analytical results from Jackson queueing network, which type should the simulation belong to? Terminating or nonterminating? Why?

(2) For the Jackson queueing network, for each station $i = 1, 2, 3$, find out the values of server utilization ρ and queue length L_Q.

(3) Build this **simplified** model in Arena, compare the output statistics with ρ and L_Q in (2), and demonstrate that the simulation model is correctly built (at least for the logical part). (*Note*: Paste the screenshot of relevant quantities you refer to; you don't need to submit this Arena file.)

Question 3 (10 points)

Now switch to our **original** problem. In the Run Setup, let number of replication be 30. Report the estimate of the expected average waiting line length in each station, together with the 95% confidence interval. (*Note*: Paste the screenshot of relevant quantities you refer to, and also submit this Arena `doe` file.)

Question 4 ($10 + 5 = 15$ points)

Based on the results in Question 3, now we want to make the half-length (i.e., half-width) of the 95% confidence interval for the expected average queue length in station 3 smaller than 0.2.

(1) How many total replications are required? (*Hint*: In Arena, the used confidence interval is $\widehat{\theta} \pm t_{n-1,1-\alpha/2}\frac{S}{\sqrt{n}}$. You can use Excel to find out the necessary quantile of t or normal distribution.)

(2) Re-run the simulation model with the number of replication calculated in (1), and show the new point estimate and confidence interval.

Question 5 (5 points)

The manager is considering to add one more server in station 3. He wants to estimate, by adding one more server in station 3, which is assumed to be identical to the current one, the reduction (in percentage) of the expected average queue length in stations 3 and 2. Use the simulation model to find out the answer.